Proceedings of BALTTRIB'2017 edited by prof. J. Padgurskas
eISSN 2424-5089/eISBN 978-609-449-127-6

https://doi.org/10.15544/balttrib.2017.27

# CHARACTERISTICS OF CONTACT OF A ROUGH SURFACE WITH A LOW-MODULUS HALF-SPACE

***P. Ogar*[1], S. Belokobylsky*, D. Gorokhov*, V. Elsukov****
* Department of Mechanical Engineering, Bratsk State University, Russia

Abstract: Initially, the contact of a single spherical asperity is considered with taking into account the influence of the remaining contacting asperities. It is assumed that the influence of the remaining contacting asperities is equal to the action of the uniform loading $q_c$ outside the asperity contour. This made possible to solve the contact problem as an axisymmetric one. An equation for the pressure distribution at the contact area is obtained. To determine the contact characteristics, a discrete roughness model is used, the surface bearing curve of which is described by a regularized beta function. The relative contact area and the gap density in the joint are determined depending on the dimensionless force elastic-geometric parameter $f_q$. When determining the gap density in the joint, the displacements of the rough surface and half-space are taken into account. It is shown that the contact characteristics do not depend on the values of the regularized beta function parameters *p* and *q*.

Keywords: rough surface, spherical asperity, elastic contact, low-modulus materials, mutual influence of asperities, relative contact area, density of gaps.

## 1. INTRODUCTION

The reliability of modern machines, units and devices is largely determined by the quality of the sealing joints. Widely used in sealing technology are coatings-form low-modulus polymer materials or individual parts [1, 2]. According to the strength criteria, structural materials are low-modulus with elastic modulus values of E < $10^3$ MPa [3]. The required tightness of the sealing joint is achieved by compressive stresses and depends on the contact characteristics – the relative contact area and the gap density [4]. Experimental studies [5] showed a slowdown in the growth of the relative contact area with the increase in load due to the mutual influence of asperities.

The purpose of this work is to theoretically substantiate the role of the mutual influence of asperities in determining the relative contact area $\eta$ and the gap density in the joint $\Lambda$.

## 2. MODEL OF THE ROUGH SURFACE

We use the discrete roughness model given in [4, 6]. Microasperities are represented by a set spherical segments of radius *r*, base $a_c$ and height $\omega R_{\max}$. The distribution of the asperities on the height corresponds to the bearing curve of real surface and defined by the regularized beta function

$$\eta(\varepsilon) = \frac{\mathrm{B}_\varepsilon(p,q)}{\mathrm{B}(p,q)}, \tag{1}$$

$\mathrm{B}_\varepsilon(p,q)$ and $\mathrm{B}(p,q)$ are the incomplete and complete beta-functions;

$$p = \left(\frac{R_p}{R_q}\right)^2 \left(\frac{R_{\max} - R_p}{R_{\max}}\right) - \frac{R_p}{R_{\max}}, \qquad q = p\left(\frac{R_{\max}}{R_p} - 1\right), \tag{2}$$

$R_p$, $R_q$, $R_{\max}$ are amplitude parameters of roughness.

[1] Author for contacts: Prof. Peter Ogar
E-mail: ogar@brstu.ru

The density of the height distribution function of the asperities is described by expression:

$$\varphi_n'(u) = \frac{u^{p-2}(1-u)^{q-2}\left[(p-1)(1-u)(q-1)u\right]}{\varepsilon_s^{p-1}(1-\varepsilon_s)^{q-1}}, \tag{3}$$

where $u$ is the relative distance from the peaks level to the peak of the $i$-th asperity, $\varepsilon_s$ is defined from condition $\varphi_n(\varepsilon_s)=1$; $\omega = 1-\varepsilon_s$.

The radius of spherical asperity is

$$r = a_c^2/(2\omega R_{max}). \tag{4}$$

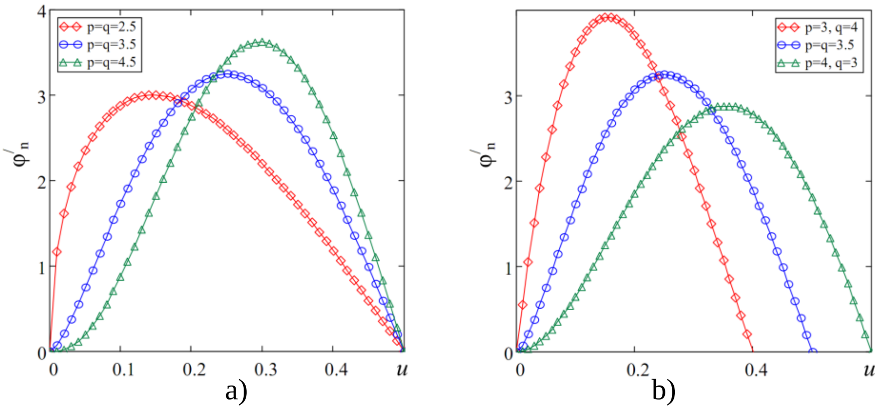


**Figure 1.** The distribution densities of asperities for different values of $p$ and $q$.

Figure 1 shows the height distribution densities of asperities for the following examples of determining the contact characteristics.

## 3. CONTACT OF A SINGLE ASPERITY AND THE ELASTIC HALF-SPACE

The contacting scheme is shown in Figure 2 [6]. In the initial position, an asperity peak is located at a distance $uR_{max}$ from the peaks line of the rough surface and the low-modulus half-space in the system of cylindrical coordinates $z$, $\rho$, $\varphi$ with origin at the point $O$ belonging to the undeformed surface of the half-space.

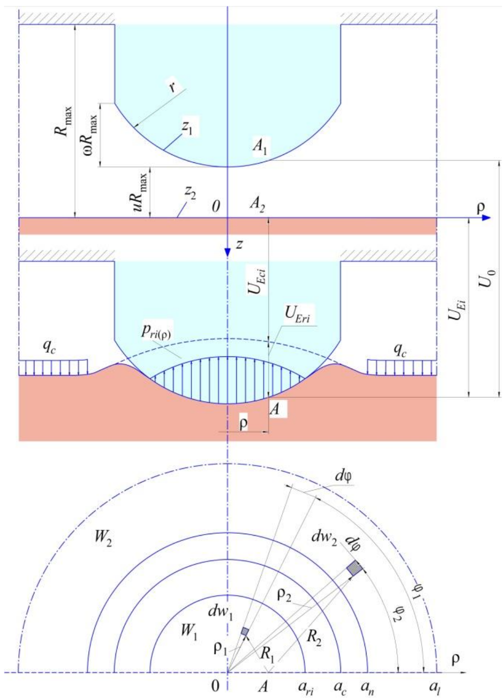

**Figure 2.** Scheme of contact of a single asperity.

After application of the compressive load, the two points $A_1$ and $A_2$ of the surface of the circular contact area $W_1(\rho = \overline{0, a_{ri}})$ come into contact. Since the general normal displacement $U_0$ of the point $A_1$ is a constant for any point of the region $W_1$, we have

$$U_0 = U_E + z_1 = U_{Eri} + U_{Eci} + z_1, \tag{5}$$

where $U_{Eri}$ is the normal contact displacement under the pressure $p_{ri}$ acting in the region $W_1$; $U_{Eci}$ is the normal displacement under the pressure $q_c$ acting in the region $W_2(\rho = \overline{a_n, a_l})$; $z_1$ is the equation of the surface of a spherical asperity in an unloaded state

$$z_1 = -uR_{\max} - \frac{\rho^2}{2r}. \tag{6}$$

We give the equations needed later to determine the gap density in the joint.

For displacements $U_{Eri}$ and $U_{Eci}$ using the data of [7], we have

$$U_{Eri} = \frac{\Theta}{\pi} \int_{W_1} \frac{p_{ri}(\rho) dw_1}{R_1}, \tag{7}$$

$$U_{Eci} = \frac{\Theta q_c}{\pi} \int_{W_2} \frac{dw_2}{R_2} = \frac{4}{\pi} \Theta q_c \left[ a_l \mathrm{E}\left(\frac{\rho_i}{a_l}\right) - a_n \mathrm{E}\left(\frac{\rho_i}{a_n}\right) \right], \tag{8}$$

where $dw_1 = \rho_1 d\rho d\varphi$, $dw_2 = \rho_2 d\rho d\varphi$; $R_j^2 = \rho^2 + \rho_j^2 - 2\rho\rho_j \cos\varphi_j$, $j = 1, 2$; $\rho \equiv \rho_i$; $\mathrm{E}(x)$ is the complete elliptic integral of the second kind.

Using the general solution of the basic equation of the axisymmetric contact problem [8] and taking into account that $\eta_i = a_{ri}^2 / a_{ci}^2$, $q_{ci} = P_i / (\pi a_{ci}^2)$ in [6] is obtained

$$p_{ri}(\rho_i) = \frac{4\eta_i^{0.5} \omega R_{\max}}{\pi \theta a_c^2} \sqrt{1 - \frac{\rho_i^2}{a_{ri}^2}} + \frac{2q_c}{\pi} \arcsin \sqrt{\frac{a_{ri}^2 - \rho_i^2}{a_c^2 - \rho_i^2}}, \tag{9}$$

$$q_{ci} = \frac{8\omega R_{\max} \eta_i^{1.5}}{3\pi\theta a_c} + \frac{2}{\pi} q_c \left[ \arcsin \eta_i^{0.5} - \sqrt{\eta_i (1 - \eta_i)} \right]. \tag{10}$$

For the mean $p_{mi}$ and the maximum $p_{ri}(0)$ stresses at the contact spot:

$$p_{mi} = \frac{N_i}{A_{ri}} = \frac{q_{ci}}{\eta_i} = \frac{8\eta_i^{0,5} \omega R_{\max}}{3\pi\theta a_c} + \frac{2q_c}{\pi \eta_i} \left[ \arcsin \eta_i^{0.5} - \sqrt{\eta_i (1 - \eta_i)} \right] \tag{11}$$

$$p_{ri}(0) = \frac{4\eta_i^{0,5} \omega R_{\max}}{\pi \theta a_c} + \frac{2q_c}{\pi} \arcsin \eta_i^{0.5}. \tag{12}$$

With sufficient accuracy (with an error of less than 1%), Eq. (9) can be written as

$$p_r(\eta_i, \rho_i) = p_{r0}(\eta_i, 0)\left(1 - \rho_i^2 / a_r^2\right)^\beta, \tag{13}$$

where $\beta = p_{r0}(\eta_i, 0) / p_m(\eta_i, 0) - 1$.

## 4. THE CONTACT OF A ROUGH SURFACE AND THE ELASTIC HALF-SPACE

The displacement of a rough surface [9]:

$$U_0 = uR_{\max} + 2\Theta q_c (a_l - a_c) + 2\omega R_{\max} \frac{a_{ri}^2}{a_c^2} + 2\theta q_c a_c \left( 1 - \sqrt{1 - \frac{a_{ri}^2}{a_c^2}} \right). \tag{14}$$

For an asperity contacting at a point, that is, for $a_{ri} = 0$, we have

$$U_0 = \varepsilon R_{\max} + 2\theta q_c (a_l - a_c). \tag{15}$$

Since the value of $U_0$ is constant for all points of the contact regions, we obtain from (14) and (15)

$$\eta_i + \frac{\theta q_c a_c}{\omega R_{\max}}\left(1 - \sqrt{1-\eta_i}\right) - \frac{\varepsilon - u}{2\omega} = 0. \tag{16}$$

This equation has a solution

$$\eta_i = \frac{\varepsilon - u}{2\omega} - f_q\left(1 + \frac{f_q}{2} - \sqrt{\left(1 + \frac{f_q}{2}\right)^2 - \frac{\varepsilon - u}{2\omega}}\right), \tag{17}$$

where $f_q = \dfrac{\theta q_c a_c}{\omega R_{\max}}$.

Taking into account Eq. (10) for a rough surface, we obtain

$$f_q(\varepsilon) = \frac{\theta q_c(\varepsilon) a_c}{\omega R_{\max}} = \frac{\frac{8}{3\pi}\int_0^{\min(\varepsilon,\varepsilon_s)} \eta_i^{1,5} \varphi_n'(u)du}{1 - \int_0^{\min(\varepsilon,\varepsilon_s)} \psi_\eta(\eta_i)\varphi_n'(u)du}, \tag{18}$$

where $\psi_\eta(\eta_i) = \dfrac{2}{\pi}\left[\arcsin \eta_i^{0.5} - \sqrt{\eta_i(1-\eta_i)}\right]$.

For the relative contact area

$$\eta(\varepsilon) = \int_0^{\min(\varepsilon,\varepsilon_s)} \eta_i \varphi_n'(u)du. \tag{19}$$

To determine the dependency $\eta(f_q)$, it is necessary to exclude the parameter $\varepsilon$ from the dependences $f_q(\varepsilon)$ and $\eta(\varepsilon)$.

Figure 3 shows the dependences of the relative contact area on the force elastic-geometric parameter $f_q$.

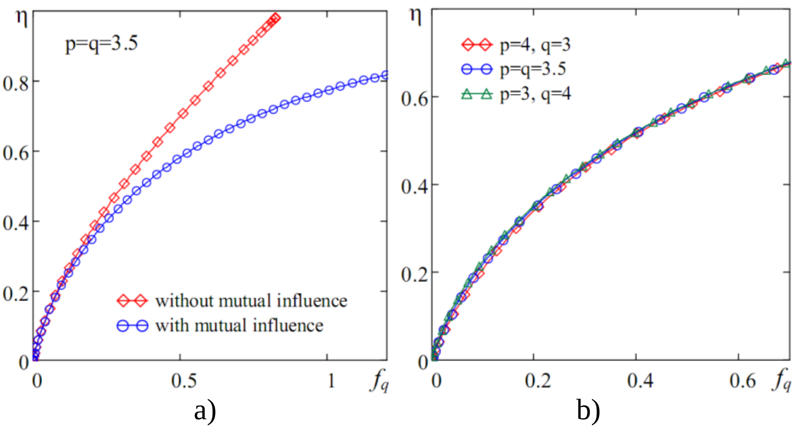


**Figure 3.** The relative contact area: with and without taking into account the mutual influence of asperities (a); for different values of $p$ and $q$ (b).

To determine the volume of the intercontact space, it is necessary to determine the volumes of gaps attributable to single contacting and noncontacting asperities [4, 9]

$$V_c = \begin{cases} V_{ri} = 2\pi \int\limits_{a_{ri}}^{a_c} \left[z_{20}(\rho) - z_{10}(\rho)\right]\rho d\rho; \\ V_{0i} = 2\pi \int\limits_{0}^{a_{ci}} \left[z_{2r}(\rho) - z_{1r}(\rho)\right]\rho d\rho, \end{cases} \tag{20}$$

where $z_{10}, z_{20} -$ the equations describing surfaces of noncontacting asperities and half-space; $z_{1r}, z_{2r} -$ the equations describing the surfaces of contacting asperities and half-space.

The density of the gaps is

$$\Lambda(\varepsilon) = \frac{V_c}{A_c R_{\max}} = \frac{1}{A_{ci} R_{\max}} \times \left[ \int\limits_{0}^{\min(\varepsilon,\varepsilon_S)} V_{ri}\varphi_n'(u)du + \int\limits_{\min(\varepsilon,\varepsilon_S)}^{\varepsilon_S} V_{0i}\varphi_n'(u)du \right]. \tag{21}$$

Taking into account that $V_{ri} = A_{ci} R_{\max} \Lambda_{ri}$ and $V_{0i} = A_{ci} R_{\max} \Lambda_{0i}$ , the Eq. (21) can be represented in the form

$$\Lambda(\varepsilon) = \int\limits_{0}^{\min(\varepsilon,\varepsilon_S)} \Lambda_{ri}\varphi_n'(u)du + \int\limits_{\min(\varepsilon,\varepsilon_S)}^{\varepsilon_S} \Lambda_{0i}\varphi_n'(u)du.. \tag{22}$$

Using the methodology of [9], we obtain

$$\Lambda_{oi} = \omega\left\{\frac{1}{2} - \frac{\varepsilon - u}{\omega} - 2 f_q \left[(k-1) - -k \cdot {}_2F_1\left(-\frac{1}{2}, \frac{1}{2}; 2; \frac{1}{k^2}\right) + + {}_2F_1\left(-\frac{1}{2}, \frac{1}{2}; 2; 1\right)\right]\right\}., \tag{23}$$

$$\begin{aligned} \Lambda_{ri} = \omega\Bigg\{(1-\eta_i)\left[\frac{1+\eta_i}{2} - \frac{\varepsilon - u}{\omega} - 2 f_q (k-1)\right] + 2 f_q k\left[{}_2F_1\left(-\frac{1}{2}, \frac{1}{2}; 2; \frac{1}{k^2}\right) - \eta_i \, {}_2F_1\left(-\frac{1}{2}, \frac{1}{2}; 2; \frac{\eta_i}{k^2}\right)\right] - \\ - 2 f_q \left[{}_2F_1\left(-\frac{1}{2}, \frac{1}{2}; 2; 1\right) - \eta_i \, {}_2F_1\left(-\frac{1}{2}, \frac{1}{2}; 2; \eta_i\right)\right] + + 2 f_{qi}\left[{}_2F_1\left(-\frac{1}{2}, \frac{1}{2}; \beta + 2; \eta_i\right) - \eta_i^{0,5} \, {}_2F_1\left(-\frac{1}{2}, \frac{1}{2}; \beta + 2; 1\right)\right]\Bigg\}, \end{aligned} \tag{24}$$

where ${}_2F_1$ is the Gauss hypergeometric function.

Substituting the expressions obtained in (22), we determine the density of the joint $\Lambda(\varepsilon)$. To determine the dependence $\Lambda(f_q)$, it is necessary to exclude the parameter from the dependences $f_q(\varepsilon)$ and $\Lambda(\varepsilon)$.

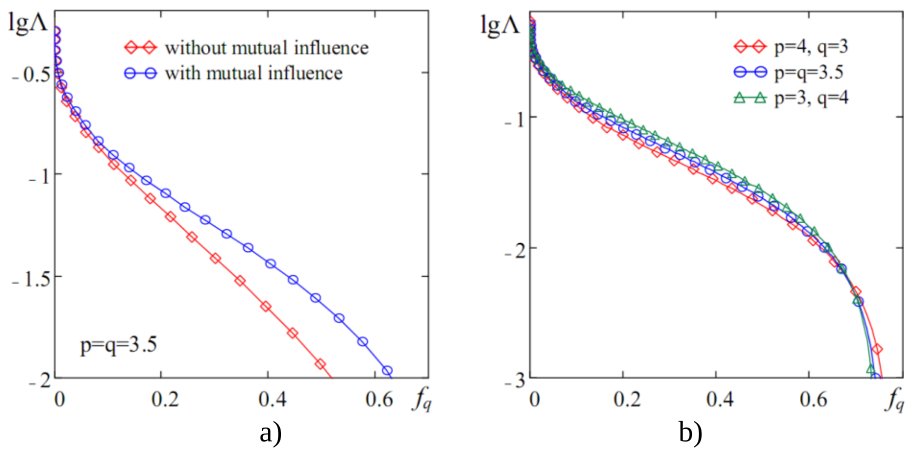


**Figure 4.** The gap density: with and without taking into account the mutual influence of asperities (a); for different values of *p* and *q* (b).

Figure 4 shows the dependence of the gap density $\Lambda$ on the complex parameter $f_q$ when two rough surfaces come into contact. From Fig. 4 it follows that $\Lambda$ depends on the influence conditions of the asperities and to a much lesser extent depends on the values of the parameters *p* and *q*.

## 5. CONCLUSIONS

The contact of an individual asperity with a low-modulus half-space is considered taking into account the influence of the remaining contacting asperities, whose action is equivalent to a uniformly distributed load $q_c$. An expression is obtained for the distribution of pressure at the contact site, the particular case of which is the Hertz distribution at $q_c = 0$.

To determine the contact characteristics, a discrete model of roughness is used, the curve of the bearing surface is described by a regularized beta function. The proposed model of roughness in a wide range describes the distribution of asperities corresponding to different bearing curves.

The contact characteristics are determined depending on the dimensionless power elastic-geometric parameter $f_q$. The mutual influence of asperities has a significant effect on the contact characteristics. The relative contact area $\eta \to 1$ at $f_q \to \infty$, which is confirmed by the empirical dependence of Bartenev-Lavrentiev [5]. The parameters of the surface bearing curve do not affect the relative contact area and have little effect on the density of gaps in the joint.

When determining the gap density in the joint, the displacements of the rough surface and the individual (for each asperity) portions of the half-space are taken into account.

## REFERENCES

[1] A. Golubev, L. Kondakov editors., Handbook of seals and sealing equipment, 2nd ed., Moscow, Mechanical Engineing,1994, 448 p. (in Russian)
[2] L. Pinchuk, A. Neverov, Polymer sealing materials, Moscow, Mechanical Engineering, 1995, 160 p.
[3] V. Goldade, A. Neverov, L. Pinchuk, Low-modulus composites based on thermoplastics, Minsk, Science and Technics, 1984, 231 p.
[4] P. Ogar, D. Gorokhov, A. Kozhevnikov, Contact problems in hermetic sealing studies of fixed joints, Bratsk, BrGU, 2017, 242 p. (in Russian)
[5] G. Bartenev, V. Lavrentiev, Friction and wear of polymers, Moscow, Chemistry, 1972, 240 p. (in Russian)
[6] P. Ogar, D. Gorokhov, V. Elsukov, Elastic contact of a rigid rough surface with the low-modulus half-space, Systems. Methods. Technologies, 2017, № 2, p. 7–12. (in Russian)
[7] S. Timoshenko, J. Goodyer, Theory of Elasticity, Moscow, Nauka, 1979, 560 p. (in Russian)
[8] I. Shtaerman, Contact problem of the theory of elasticity, Moscow–Leningrad, Gostekhizdat, 1949, 270 p. (in Russian)
[9] P. Ogar, D. Gorokhov , A. Kozhevnikov, The density of gaps in the seal joint in elastic contact of microasperities, Proc. 2nd Int.l Conf " Modelling, Identification and Control", Paris, 9–10.04. 2015. p. 177–180.