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EFFECT OF THE TEMPORAL PROFILE OF THE FRICTION POWER ON THERMAL STRESSES DURING BRAKING

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Abstract: In this paper influence of the temporal profile of frictional heat flow density on the distributions of temperature and thermal stresses in a friction element during single braking was investigated. For this purpose a one-dimensional boundary-value heat conduction problem for a half-space body (which simulates a brake rotor) heated on the outer surface by the heat flux with different intensities was formulated and solved by means of Duhamel's theorem. Solutions were obtained for ten temporal profiles of specific friction power, which are proportional to the intensity of frictional heat flow. Based on received transient temperature fields and Timoshenko's model of thermal bending of a thick plate with unfixed edges, the analytical distributions of quasi-static thermal stresses in a friction element were found. Achieved solutions allow conducting numerical analysis of the distributions of temperature and thermal stresses in a brake rotor with different time profiles of the specific friction power.

Keywords: temperature, thermal stresses, time-dependent frictional heat flow, braking.

1. INTRODUCTION

On the friction surface in a disc-pad system during braking, thermal energy is generated due to friction. Frictional heat fluxes penetrate the friction elements and heat it. As a result, brake disc and pad are exposed to high temperature and stresses during exploitation. In these conditions, the rate of wear increases exponentially and the braking effectiveness lowers. Exceeding of the ultimate strength of the friction material by the value of thermal stress can cause plastic deformations and thermal cracks on the working surface of a brake disc. Therefore, thermal calculations of the braking system are the main stage of a brake design process.

Frictional heating process is often analytically modeled based on the common assumptions of the linear heat conduction theory and thermoelasticity. In order to predict values of temperature and stresses in a designed friction node, frictional heat conduction problem for bodies with canonical shape, where e.g. friction element is replaced by a half-space, are formulated [1–4]. This replacement can be applied, because during rapid heating processes (e.g. single braking of short duration) changes of the temperature and thermal stresses occur in material only on certain distance from the friction surface. However, a unidirectional model of heat conduction is legitimate for high values of Peclet's number (for hard braking processes) [5].

Common approach to model the frictional heating process is virtual separation of the friction elements and independent heating their friction surfaces with the heat fluxes, which densities are proportional to the friction power (i.e. product of friction coefficient, contact pressure and relative sliding speed) [6]. Experimental researches show that in real braking processes all these quantities significantly change with time, and the character of these changes depends on working conditions, thermophysical properties of materials and brake construction etc. Consequently the specific power of friction takes different temporal profiles for different braking modes. Despite that, most of exact solutions to the frictional heat conduction problem were obtained with simplified form of heat flux intensity, i.e. constant or linearly decreasing function of time. This corresponds to the processes with constant

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velocity or constant deceleration, with assumption of invariable values of pressure and the coefficient of friction.

Analytical solution to a one-dimensional heat conduction problem for a semi-infinite body, heated on the working surface by the heat flow with intensity, which is invariable under time, was obtained in the monograph [7]. Corresponding thermal problem of friction for a homogeneous half-space, heated on the surface by the frictional heat flow with linearly decreasing friction power was considered in the article [8]. In the paper [4] fields of the temperature and normal, thermal stresses in a half-space heated on its outer surface by the heat flux modeled by a pulse with rectangular or triangular shape, were determined. Frictional heat conduction problem for a caliper and pad simulated by the half-space – strip system, with linearly reducing density of the frictional heat flux, which effects on the outer surface of the strip was solved in the article [9].

The aim of this work is to investigate the effect of the selected temporal profiles of specific friction power on the distributions of the thermal stresses and temperature in the friction element during single braking. For this purpose we formulated and solved a heat conduction problem of friction for a homogeneous half-space (which simulates a brake rotor), heated on the friction surface by the heat flux with different intensities. We obtained solutions for ten time profiles of specific friction power, which are proportional to the intensities of the frictional heat fluxes. These profiles were established based on experimental research for different braking modes in the monograph [10]. Based on received solutions (transient temperature fields) and model of thermal bending of a thick strip with unfixed ends [11], we determined analytical distributions of the quasi-static normal thermal stresses. Then, we carried out numerical analysis of the dimensionless distributions of temperature and thermal stresses in a brake rotor during single process of braking with different profiles of friction power.

2. STATEMENT AND SOLUTION TO THE PROBLEM

Experimentally determined temporal profiles of specific friction power during single braking have the following forms [10]:

$$q_i(t) = q_0 q_i^*(t),\ q_0 = w_0 / t_s,\ 0 \le t \le t_s,\ i = \overline{1,10}, \tag{1}$$

$$q_1^*(t) = 2(1-t^*),\ q_2^*(t) = 2t^*,\ q_3^*(t) = 1.5\sqrt{1-t^*},\ q_4^*(t) = 1.5\sqrt{t^*}, \tag{2}$$

$$q_5^*(t) = 3(1-t^*)^2,\ q_6^*(t) = 3t^{*2},\ q_7^*(t) = 6t^*(1-t^*),\ q_8^*(t) = 1.2(1-t^*)(1+2t^*), \tag{3}$$

$$q_9^*(t) = 1.2t^*(3-2t^*),\ q_{10}^*(t) = 6\sqrt{t^*}(1-\sqrt{t^*}),\ t^* = t/t_s, \tag{4}$$

where $$w_0 = \int_0^{t_s} q_i(t)dt,\ i = \overline{1,10}, \tag{5}$$

w_0 is a constant braking work, which is made by the braking system at the time $t = t_s$.

Taking into account the above mentioned assumptions, the dimensionless temperature T_i^*, that is initiated by heating the outer surface $z = 0$ of the semi-infinity body $z \ge 0$ by heat fluxes with intensity q_i, $i = \overline{1,10}$ (1), we find from solution to the following heat conduction problem of friction:

$$\frac{\partial^2 T_i^*(\zeta,\tau)}{\partial \zeta^2} = \frac{\partial T_i^*(\zeta,\tau)}{\partial \tau},\ \zeta > 0,\ 0 < \tau \le \tau_s,\ i = \overline{1,10}, \tag{6}$$

$$\left.\frac{\partial T_i^*(\zeta,\tau)}{\partial \zeta}\right|_{\zeta=0} = -q_i^*(\tau),\ 0 < \tau \le \tau_s,\ i = \overline{1,10}, \tag{7}$$

$$T_i^*(\zeta,\tau) \to 0,\ \zeta \to \infty,\ 0 < \tau \le \tau_s,\ i = \overline{1,10}, \tag{8}$$

$$T_i^*(\zeta,0)=0,\ \ \zeta \geq 0\,,\ i=\overline{1,10}\,, \tag{9}$$

where $$\zeta=\frac{z}{a}\,,\ \tau=\frac{kt}{a^2},\ \tau_s=\frac{kt_s}{a^2}\,,\ T_i^*=\frac{T_i-T_a}{T_0}\,,\ T_0=\frac{q_0 a}{K}, \tag{10}$$

$a=\sqrt{3kt_s}$ – effective depth of the heat penetration [12], k – coefficient of thermal diffusivity, K – coefficient of thermal conductivity, T_a – ambient temperature.

The solution to the heat conduction problem (6)–(9) will be found using the Duhamel's formula [13]:

$$T_i^*(\zeta,\tau)=\int_0^{\tau} q_i^*(s)\frac{\partial}{\partial\tau}T_0^*(\zeta,\tau-s)ds\,,\ \zeta\geq 0\,,\ 0\leq\tau\leq\tau_s\,,\ i=\overline{1,10}\,, \tag{11}$$

where [7] $$T_0^*(\zeta,\tau)=2\sqrt{\tau}\,\mathrm{ierfc}\,Z\,,\ \zeta\geq 0\,,\ 0\leq\tau\leq\tau_s\,, \tag{12}$$

T_0^* is the known solution to the same problem with the function $q_0^*(\tau)=1$ in the boundary condition (7) and $Z=0.5\zeta/\sqrt{\tau}$.
Solutions to the problem for ten profiles of specific friction power (1)–(4) were found in the form [14]:

$$T_1^*(\zeta,\tau)=4\sqrt{\tau}\ \tau^*\{[3/\tau^*-2(1+Z^2)]\mathrm{ierfc}\,Z+Z\mathrm{erfc}\,Z\}/3\,, \tag{13}$$

$$T_2^*(\zeta,\tau)=4\sqrt{\tau}\,\tau^*[2(1+Z^2)\mathrm{ierfc}\,Z-Z\mathrm{erfc}\,Z]/3\,, \tag{14}$$

$$\begin{aligned}T_3^*(\zeta,\tau)=&\,3\sqrt{\tau}\,\mathrm{ierfc}Z-0.5\sqrt{\tau}\,\tau^*[2(1+Z^2)\mathrm{ierfc}\,Z-Z\mathrm{erfc}\,Z]-\\&-\sqrt{\tau}\,\tau^{*2}[(8+18Z^2+4Z^4)\mathrm{ierfc}\,Z-Z(7+2Z^2)\mathrm{erfc}\,Z]/40-\\&-\sqrt{\tau}\ \tau^{*3}[(48+174Z^2+80Z^4+8Z^6)\mathrm{ierfc}\,Z-Z(57+36Z^2+4Z^4)\mathrm{erfc}\,Z]/560,\end{aligned} \tag{15}$$

$$T_4^*(\zeta,\tau)=0.75\sqrt{\pi\tau_s}\,\tau^*[\mathrm{erfc}\,Z-2Z\,\mathrm{ierfc}\,Z], \tag{16}$$

$$\begin{aligned}T_5^*(\zeta,\tau)=&\,2\sqrt{\tau}\{3\mathrm{ierfc}\,Z-2\tau^*[2(1+Z^2)\mathrm{ierfc}\,Z-Z\mathrm{erfc}Z]+\\&+0.2\,\tau^{*2}[(8+18Z^2+4Z^4)\mathrm{ierfc}Z-Z(7+2Z^2)\mathrm{erfc}Z]\},\end{aligned} \tag{17}$$

$$T_6^*(\zeta,\tau)=0.4\sqrt{\tau}\ \tau^{*2}[(8+18Z^2+4Z^4)\mathrm{ierfc}\,Z-Z(7+2Z^2)\mathrm{erfc}\,Z], \tag{18}$$

$$\begin{aligned}T_7^*(\zeta,\tau)=&\,4\sqrt{\tau}\,\tau^*\{[2(1+Z^2)\mathrm{ierfc}\,Z-Z\mathrm{erfc}\,Z]-\\&-0.2\tau^*[(8+18Z^2+4Z^4)\mathrm{ierfc}\,Z-Z(7+2Z^2)\mathrm{erfc}\,Z]\},\end{aligned} \tag{19}$$

$$\begin{aligned}T_8^*(\zeta,\tau)=&\,0.8\sqrt{\tau}\{3\mathrm{ierfc}\,Z+\tau^*[2(1+Z^2)\mathrm{ierfc}\,Z-Z\mathrm{erfc}Z]-\\&-0.4\tau^{*2}[(8+18Z^2+4Z^4)\mathrm{ierfc}\,Z-Z(7+2Z^2)\mathrm{erfc}\,Z]\},\end{aligned} \tag{20}$$

$$\begin{aligned}T_9^*(\zeta,\tau)=&\,0.8\sqrt{\tau}\,\tau^*\{3[2(1+Z^2)\mathrm{ierfc}\,Z-Z\mathrm{erfc}\,Z]-\\&-0.4\tau^*[(8+18Z^2+4Z^4)\mathrm{ierfc}\,Z-Z(7+2Z^2)\mathrm{erfc}\,Z]\},\end{aligned} \tag{21}$$

$$T_{10}^*(\zeta,\tau)=\tau^*\{3\sqrt{\pi\tau_s}(\mathrm{erfc}\,Z-2Z\,\mathrm{ierfc}\,Z)-4\sqrt{\tau}[2(1+Z^2)\mathrm{ierfc}\,Z-Z\mathrm{erfc}\,Z]\}, \tag{22}$$

where $\tau^*=\tau/\tau_s$.

At establishing the field of dimensionless temperature $T_3^*(\zeta,\tau)$ (15), the expansion of the function $q_3^*(\tau)$ in the following form, was used:

$$q_3^*(\tau) = 1.5[1 - 0.5\tau^* - 0.125\tau^{*2} - 0.0625\tau^{*3} + ...], \quad (23)$$

with the restriction of series to the three first terms of this series.

Thermal stresses $\sigma_i^*(\zeta,\tau)$, corresponding to the temperature fields $T_i^*(\zeta,\tau)$ (13)–(22), we received using the following model of thermal bending of a thick plate with unfixed edges [11]:

$$\sigma_{i,x}(z,t) = \sigma_{i,y}(z,t) \equiv \sigma_i(z,t)\,,\ \sigma_{i,z}(z,t) = 0 \quad (24)$$

$$\sigma_i(z,t) = \sigma_0\sigma_i^*(\zeta,\tau)\,, \sigma_0 = \alpha_t ET_0/(1-\nu)\,,\ 0 \le z \le a\,,\ 0 \le t \le t_s\,,\ i = \overline{1,10}\,, \quad (25)$$

$$\sigma_i^*(\zeta,\tau) = \varepsilon_i^*(\zeta,\tau) - T_i^*(\zeta,\tau)\,,\ 0 \le \zeta \le 1,\ 0 \le \tau \le \tau_s\,, \quad (26)$$

$$\varepsilon_i^*(\zeta,\tau) = (4-6\zeta)N_i(\tau) + 6(2\zeta-1)M_i(\tau)\,,\ N_i(\tau) = \int_0^1 T_i^*(\zeta,\tau)d\zeta\,,\ M_i(\tau) = \int_0^1 \zeta T_i^*(\zeta,\tau)d\zeta\,. \quad (27)$$

where ε_i^* are dimensionless temperature strains, $N_i(\tau)$ – averaged over the plate thickness temporal profiles of temperature and $M_i(\tau)$ – the temperature momentum.

3. NUMERICAL ANALYSIS

The dimensionless input parameters used to conduct numerical analysis are: the distance from the heated surface ζ , time τ and stop time $\tau_s = 1/3$ (10).

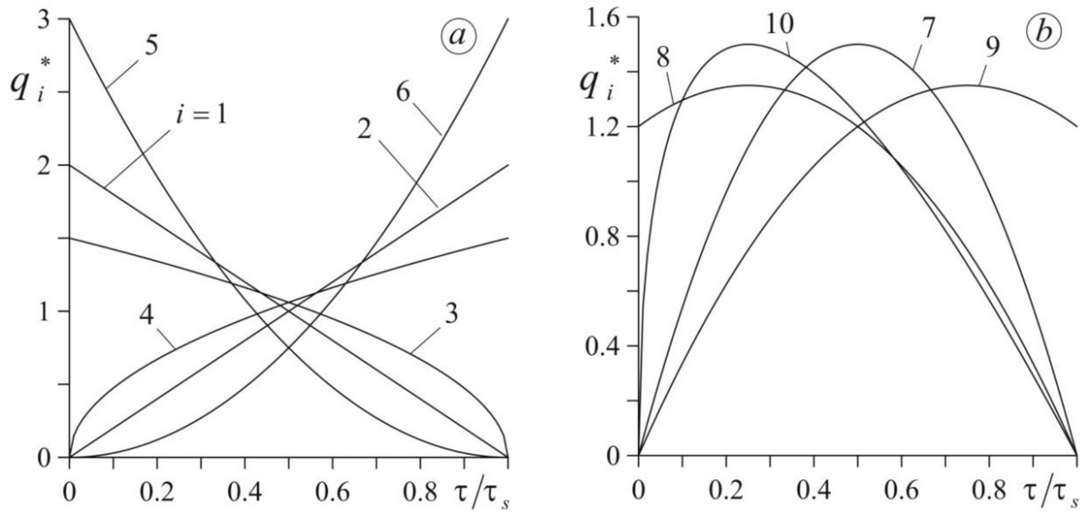

Figure 1. Evolutions of the dimensionless densities of friction power $q_i^*(\tau)$: a) $i = \overline{1,6}$; b) $i = \overline{7,10}$.

The profiles of the densities of friction power (Fig. 1), can be categorized into three groups. The first one contains cases *i* = 1, 3, 5, which reach maximum values at the initial moment $\tau = 0$, and then monotonically decrease to zero at the moment of standstill (Fig. 1 a). The graph of the function $q_1^*(\tau)$ changes linearly, and the corresponding dimensionless temperature reaches its maximum value $T_1^* = 0.61$ in the middle of the braking time at $\tau = 0.5\tau_s$ (Fig. 2 a), which is distinctive for braking with constant deceleration [15]. The curves *i* = 3.5 have parabolic shapes (concave at *i* = 3 and convex at *i* = 5), which decrease from the maximum $q_i^* = 1.5,3$, *i* = 3.5 at $\tau = 0$ to zero at the stop $\tau = \tau_s$ (Fig. 1 a). The maximum temperature $T_i^* = 0.59, 0.69$ at *i* = 3.5 are reached at the moments of time

$\tau_i = 0.74\tau_s, 0.32\tau_s$, i = 3.5 (Fig. 2 a). The dimensionless thermal stresses σ_i^*, i = 1, 3, 5 on the friction surface are demonstrated in Fig. 3 a. After start of the braking, stresses are compressed, and its absolute values rapidly increase, achieving maximum $\sigma_i^* = 0.18, 0.15, 0.26$ at moments $\tau_i = 0.1\tau_s$, $0.12\tau_s, 0.08\tau_s$ for i = 1, 3, 5, accordingly (Fig. 3 a). Then, the compressive stresses in cases i = 1.5 on the heated surface disappear with time, and at the time moments $0.85\tau_s$ $(i = 1)$, and $0.72\,\tau_s\,(i = 5)$ change the sign of stresses and then tensile stresses appear.

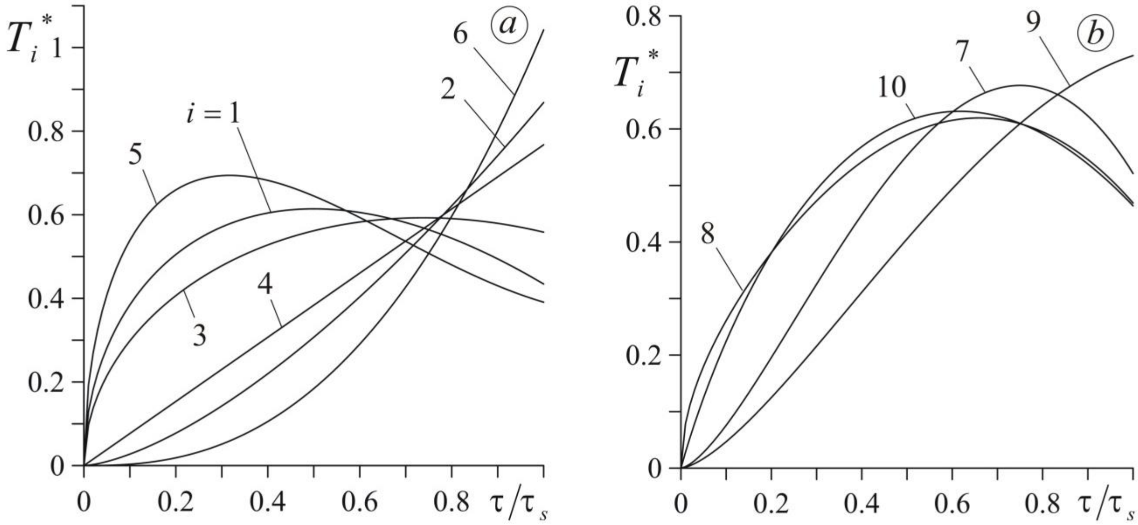

Figure 2. Evolutions of dimensionless temperature $T_i^*(0,\tau)$ on the heated surface: a) $i = \overline{1.6}$; b) $i = \overline{7.10}$.

The second group comprises curves with numbers i = 2, 4, 6 (Fig. 1 a). These functions monotonically rise from zero value at the moment $\tau = 0$ to the maximum values $q_i^* = 2, 1.5, 3$, i = 2, 4, 6, at the stop moment. Similarly, corresponding dimensionless temperatures $T_i^*(0,\tau)$, i = 2, 4, 6 (Fig. 2 a) change with time. The maximum values of these temperatures at the standstill are $T_{i,}^* = 0.87, 0.77, 1.04$ for i = 2, 4, 6, respectively. Corresponding thermal stresses σ_i^*, i = 2, 4, 6 on the outer surface are compressed during the whole braking, and its absolute values monotonically increase with time from zero at the start moment to the maximum $\sigma_i^* = 0.17,\ 0.12,\ 0.27$, i = 2, 4, 6, respectively, at the stop time (Fig. 3 a).

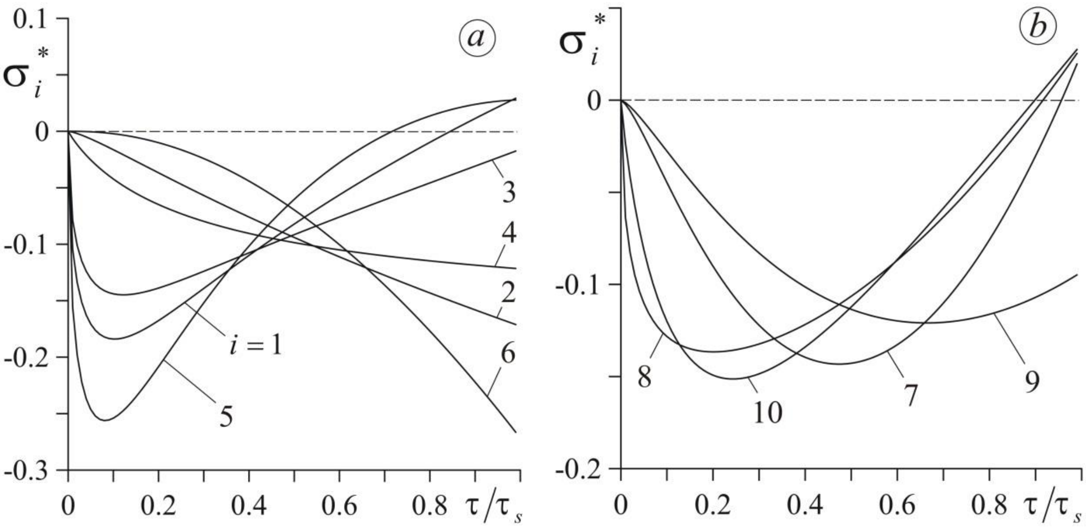

Figure 3. Evolutions of dimensionless thermal stresses $\sigma_i^*(0,\tau)$ on the heated surface: a) $i = \overline{1.6}$; b) $i = \overline{7.10}$.

The third group includes the functions $i = \overline{7.10}$ are presented in Fig. 1 b. These curves have a local maximum within a time braking interval $0 \le \tau \le \tau_s$. The values of local maximum of the profiles $q_i^*(\tau)$, $i=\overline{7,10}$, are reached at time moments $\tau_i = 0.5\tau_s, 0.25\tau_s, 0.75\tau_s, 0.25\tau_s$, respectively. The corresponding dimensionless temperatures $T_i^*(0,\tau)$ (Fig. 2b) also increase with the beginning of process until they achieve the maximum $T_i^* = 0.68, 0.62, 0.73, 0.63$, $i=\overline{7.10}$. Then, in the cases i = 7, 8, 10 cooling of the working surface follows, but in case i = 9 the temperature increases till the standstill. Temporal profiles of specific friction power effects on evolution of the corresponding stresses, which are presented in the Fig. 3 b. In the initial period of braking, the absolute values of compressive stresses decrease, reaching local minimum $\sigma_i^* = -0.14, -0.14, -0.12, -0.15$ at $\tau = 0.47\tau_s, 0.2\tau_s, 0.66\tau_s, 0.24\tau_s$, i = 7, 8, 9, 10, respectively (Fig. 3b). Stresses σ_i^*, i = 7, 8, 10 at the time moments $\tau = 0.96\tau_s, 0.92\tau_s, 0.90\tau_s$, accordingly, change the sign, and its highest values $\sigma_i^* = 0.02, 0.03, 0.02$ are achieved at the stop moment. In case i = 9, stresses on the friction surface of the element compress the material during the whole braking process.

4. CONCLUSIONS

It has been established, that the shape of the temporal profile of specific friction power has a significant influence both on the evolution of temperature and thermal stresses on the heated surface of the frictional element. Conducted analysis shows that the highest values of the superficial temperature are proportional to the highest values of specific friction power. The greater the time to achieve the highest value of specific friction power, the greater the time for temperature to reach its maximum value.

A close relation between the evolution of the thermal stresses and the maximum value of the densities of friction power was observed. The time of achieving the maximum friction power has mainly influence on the moment of thermal stresses sign change on the working surface. In case of exceed of the ultimate strength of the friction material by the value of tensile stress can cause the initiation of the thermal cracks on working surface [4].

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