FORMS OF $\varphi_x - s_x$ – DIAGRAMS OF AN AUTOMOBILE TYRE

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Abstract: Vehicle stability, handling and braking properties significantly depend on the friction interaction between a tyre and the road surface. Index of the friction interaction is the coefficient of friction in different coordinates. The friction coefficient is generally calculated as a function of the coefficient of the longitudinal slip of the wheel $\varphi_x = f(s_x)$. The lateral force significantly affects the coefficient of friction. In different cases the lateral force can occur either before or after occurrence of the braking moment on the wheel. The purpose of the study is to investigate the effect of different sequence of occurrence of the lateral force and the braking moment on a wheel on the friction properties of the tyre with a solid road surface. The author have developed the methods which allows considering the sequence of occurrence of the lateral force and the moment on the wheel in the calculation of $\varphi_x - s_x$ – diagrams.

Keywords: the coefficient of friction, the sequence of occurrence of the lateral force and braking moment on the wheel, forms of $\varphi_x - s_x$– diagrams.

INTRODUCTION AND RELEVANCE

Properties of active vehicle safety, to a significant extent, depend on the friction interaction of the tyre with the supporting surface. Index of the friction interaction is the coefficient of friction [1–12]. Its minimum value is zero; its maximum value is the coefficient of static friction. It can also accept other values, one of which is the coefficient of sliding friction. Since the experimental determination of the values of the coefficients of friction is a complex process for a variety of reasons, therefore, in different tasks they are calculated. For the same road and tyre the friction coefficient is generally calculated as a function of the coefficient of the longitudinal slip of the wheel $\varphi_x = f(s_x)$. This dependency is called $\varphi_x - s_x$ – diagram. At present there are two groups of methods for the calculation of the coefficient of friction: without taking into account the lateral force of the wheel and with taking into account the lateral force. Existing calculation and experimental methods which do not consider the lateral force, are very approximate, are not having theoretical justification, and based on the processing results of a particular experiment for a particular tyre. Since most modes of motion of any wheeled vehicle are associated with the presence of the lateral force, its presence in the mathematical models for calculation of the coefficients of friction is required, because the lateral force significantly affects the coefficient of friction. In different cases the lateral force can occur either before or after occurrence of the braking moment on the wheel. The calculation–experimental method of H.B. Pacejka includes the lateral force [6, 12]. His famous “magic” formula is built on a generalization of the dependences, obtained on the basis of numerous experiments, and allows calculating $\varphi_x - s_x$ – diagrams only for the cases of exposure of the lateral force, which already existed before the onset of braking.

Earlier, the author have initially proposed two theoretical models for the calculation of $\varphi_x - s_x$ – diagrams, considering the lateral force of the wheel, obtained on the basis of hypotheses about the

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implementation of the shares of the coefficient of friction by corresponding zones in the contact patch [10]. The first one is based on the construction of $\Phi_x - s_x$ nomogram, and the second one is based on the calculation of the $CUSF$ ($CUSF$ – coefficient of use of static friction). Thus, in both cases, the sequence of occurrence of the lateral force and the moment on the wheel were not mentioned. Accordingly, the possibilities of applying the existing models in terms of the lateral force effect were not proved.

**PURPOSE:** To investigate the effect of different sequence of occurrence of the lateral force and the braking moment on a wheel on the friction properties of the tyre with a solid road surface.

Accepted designations:

- $P_z$ – normal loading of a wheel; $P_y$ – lateral force of a wheel; $R_y$ – lateral reaction of a basic surface; $M_b$ – braking moment; $V_x$ – linear speed of the axis of a wheel; $\omega$ – angular speed of a wheel; $R_k$ – dynamic radius of a wheel; $s_x$ – coefficient of the longitudinal wheel sliding; $(s_x = (V_x - \omega \cdot R_k)/V_x)$; $\alpha$ – side sliding angle (the deviation angle of the vector of the translational speed of the wheel from the longitudinal central axis upon the occurrence of the lateral slip of the wheel); $\delta$ – side slip angle (the deviation angle of the vector of the translational speed of the wheel from the longitudinal central axis up to the occurrence of the lateral slip of the wheel); $\Phi_x$ – friction coefficient; $\Phi_{x0}$ – friction coefficient at the absence of lateral force; $\Phi_{xsl}$ – share of friction coefficient, realized by a zone with static friction in contact patch; $\Phi_{xst}$ – share of friction coefficient, realized by a zone with sliding friction in contact patch; $f_{st}$ – static friction coefficient; $f_{sl}$ – share of static friction coefficient in the longitudinal direction; $f_{stl}$ – share of static friction coefficient in the lateral direction; $f_{slx}$ – share of sliding friction coefficient in the longitudinal direction; $f_{sly}$ – share of sliding friction coefficient in the lateral direction; $CUSF$ – coefficient of use of static friction; $CPSF$ – coefficient of proportionality of static friction.

**PROBLEM STATEMENT**

The lateral force of a wheel can exist prior to the beginning of the increase of the braking moment on a wheel or occur later.

Since the coefficient of friction is always realized by zones of sliding friction and static friction in the contact patch [3, 5, 6], then:

$$\Phi_x = \Phi_{xsl} + \Phi_{xst}.$$

Since at the preservation of stability of the wheel the sliding exists only in the longitudinal direction, it is always fulfilled

$$\Phi_{xsl} = f_{stl} \cdot s_x.$$

To solve the problem of the effect of the sequence of occurrence of the lateral force and the moment on the wheel on the friction properties of the tyre with the solid surface, and for the defining of this effect, at the calculation of the value of $\Phi_{xsl}$ it is necessary to proceed from the sequence of occurrence of the lateral force and the braking moment on a wheel.
ESSENCE OF PROBLEM SOLVING.

1. Determination of $\varphi_{xst}$ by the advanced author method based on $CUSF$, under different conditions of occurrence of the lateral force

Earlier it has been established:

$$\varphi_{xst} = f_{stx}(1 - s_x^*) \cdot CUSF,$$

where $CUSF = f(s_x^*)$.

For all kinds of non-studded tyres, for all kinds and conditions of the road surface [5]

$$CUSF = a + b \cdot s_x + c \cdot s_x^2 + d \cdot s_x^3 + e \cdot s_x^4 + f \cdot s_x^5 + g \cdot s_x^6 + h \cdot s_x^7 + i \cdot s_x^8 + j \cdot s_x^9,$$

where $a, b, c, d, e, f, g, h, i, j$ – constant coefficients ($a = 0.001814$; $b = 26.747630$; $c = -324.5414748$; $d = 2137.928850$; $e = -8375.670586$; $f = 20260.588666$; $g = -30442.879724$; $h = 27611.479368$; $i = -13822.359721$; $j = 2929.70553$).

For all kinds of studded tyres, for all kinds and conditions of ice [7]:

$$CUSF = \frac{a \cdot b + c \cdot s_x^d}{b + s_x^d},$$

where $a, b, c, d$ – constant coefficients ($a = 0.0006$; $b = 0.5654$; $c = 1.5740$; $d = 0.2766$).

However, earlier, the algorithm of definition of $f_{stx}$ in the formula (1) has not been proved.

It is now established, that at defining of the value of $f_{stx}$ in the formula (1) it is necessary to proceed from the sequence of occurrence of the lateral force and the braking moment on a wheel. The presence of the lateral force before and after the beginning of the braking defines the opportunities of a zone with static friction in contact patch for the realization of the friction properties of the tyre.

1.1. The lateral force existed before the onset of the braking of a wheel

At the absence of the braking ($M_b = 0$) in the contact patch there is only a zone with static friction and the existing lateral force $P_y$ is perceived by entire contact patch on which the load $P_z$ falls. In response to $P_y$ the lateral reaction occurs, $R_y = P_z \cdot f_{sty}$, where $f_{sty} = P_y / P_z$.

Since, $f_{st} = \sqrt{f_{stx}^2 + f_{sty}^2}$, then the contact patch can also realize $f_{stx} = \sqrt{f_{st}^2 - (P_y / P_z)^2}$.

1.2. The lateral force has occurred after the onset of braking of a wheel

This can be expressed by:

– a primary condition: $M_b \neq 0$; $P_y = 0$;

– a secondary condition: $M_b \neq 0$; $P_y \neq 0$.

At the presence of the braking ($M_T \neq 0$) in the contact patch there are zones both with static friction, and sliding friction. The occurred lateral force $P_y$ force is perceived only by a zone of contact patch
with static friction on which the load \( P_z(1-s_x) \) falls. In response to \( P_y \) the lateral reaction occurs \( R_y = P_z(1-s_x) \cdot f_{sty} \), where \( f_{sty} = P_y/(P_z(1-s_x)) \).

Since, \( f_{st} = \sqrt{f_{stx}^2 + f_{sty}^2} \), then the contact patch can also realize

\[
R_y = P_z(1-s_x) \cdot f_{stx} \cdot f_{sty},
\]

where \( f_{stx} = \sqrt{f_{st}^2 - (f_{sty}^2)} \).

2. Determination of \( \varphi_{xst} \) by the new author method based on CUSF, under different conditions of occurrence of the lateral force

Before the occurrence of the lateral force on a wheel and before the occurrence of the braking of a wheel in the contact patch there was only a zone with static friction, defined as \( f_{sl} \).

The occurred lateral force (before and after the beginning of the braking) uses a share of static friction \( f_{sty} \) for the realization of the lateral reaction of a basic surface.

For the realization of the friction properties in the longitudinal direction a share \( f_{stx} \) remained. It is less, than it was, by \( k_y \) times.

We will call the coefficient of proportionality of static friction (CUSF) \( k_y \) – the ratio of a share of the static friction in the contact patch, remained for the realization of the friction properties in the longitudinal direction, to the static friction coefficient.

\[
k_y = \frac{\sqrt{f_{st}^2 - f_{sty}^2}}{f_{st}}.
\]

The numerator in the formula (2) is \( f_{stx} \). In other words, CUSF \( k_y \) shows by how many times the share of static friction in contact patch has decreased at the presence of the lateral force.

**Hypothesis**: the share of friction coefficient, realized by a zone with static friction in the contact patch at the presence of the lateral force, is equal to multiplication of this value at the absence of the lateral force by the coefficient of proportionality of static friction.

This can be written as:

\[
\varphi_{xst} = (\varphi_{x0} - f_{sl} \cdot s_x) \cdot k_y.
\]

At defining of the value of \( f_{sty} \) in the formula (1) it is necessary to proceed from the sequence of occurrence of the lateral force and the braking moment on a wheel. The presence of the lateral force before and after the beginning of the braking defines the opportunities of a zone with static friction in contact patch for the realization of the friction properties of the tyre.

2.1. The lateral force existed before the onset of the braking of a wheel

At the presence of braking \( (M_b = 0) \) in the contact patch there is only a zone with static friction and the existing lateral force \( P_y \) is perceived by entire contact patch on which the load \( P_z \) falls. In response to \( P_y \) the lateral reaction occurs, \( R_y = P_z \cdot f_{sty} \), where \( f_{sty} = P_y/P_z \).

2.2. The lateral force has occurred after the onset of braking of a wheel
At the presence of braking \((M_b \neq 0)\) in the contact patch there are zones both with static friction, and sliding friction. The occurred lateral force \((P_y)\) is perceived only by a zone of contact patch with static friction on which the load \(P_z(1-s_x)\) falls. In response to \(P_y\) the lateral reaction occurs, \(R_y = P_z(1-s_x) \cdot f_{sty}\), where \(f_{sty} = P_y/P_z(1-s_x)\).

At the absence of the lateral force \(k_y = 1\) and \(\varphi_x = \varphi_{x0}\).

Figures 1 and 2 give comparison of estimated \(\varphi_x - s_x\) – diagrams, received under different conditions of occurrence of the lateral force. Fig. 1 and 2 show that the results of both calculation methods of \(\varphi_x - s_x\) – diagrams based on CUSF, and CPSF coincide with each other, under the same conditions of occurrence of the lateral force and braking moment on the wheel. Herewith the diagrams of Fig. 2 are experimentally proved while testing automobiles on the automobile practice ground, using control–measuring equipment “Corrsys Datron” [10]. To prove the validity of the diagrams of Fig. 1, the analysis of the experimental data by H. B. Pacejka has been conducted [12]. In his experimental formulas, the lateral force is not present, but there is its corresponding slip angle. Therefore, for comparison of the results of the calculating methods, offered by the author of the article, the lateral force was calculated from the slip angle of the deformation theory.

It can be seen that different sequence of occurrence of the lateral force and the braking moment on the wheel gives different “pictures” of the process.

Considering different sequence of occurrence of the lateral force and the braking moment in the calculations of \(\varphi_x - s_x\) – diagrams, regardless of the applicable methods, allows obtaining very close results of the calculation of the coefficients of friction.

**Figure 1.** Comparison of estimated \(\varphi_x - s_x\) – diagrams for the case, when the lateral force existed before the onset of the braking, at different values of \(P_y/P_z\): 1 – by the method, based on CPSF; 2 – by the method, based on CUSF.
Figure 2. Comparison of estimated $\Phi_X - s_X$ - diagrams for the case, when the lateral force has occurred after the onset of the braking, at different values of $P_y / P_z$: 1 – by the method, based on $CPSF$; 2 – by the method, based on $CUSF$.

CONCLUSIONS:

- It is found that there is a significant effect of different sequence of occurrence of the lateral force and the moment on the wheel on the friction properties of zones of static friction and sliding friction in the contact patch of a tyre with the road, as well as on the location of the maximum of the $\Phi_X - s_X$ – diagram.

- The universal methods were developed, which allow considering the sequence of occurrence of the lateral force and the braking moment on the wheel when calculating the $\Phi_X - s_X$ – diagram: $CUSF$ and $CPSF$. The methods are universal for all the types of tyres, types and conditions of road surface. Along with this the method, based on the $CPSF$, also allows obtaining more accurate results, if the zero $\Phi_X - s_X$–diagram is known for a specific tyre and the road.

- It is found that if the lateral force existed before the beginning of the braking of the vehicle (braking when turning the vehicle, etc.), then in case of its increase the maximum of the coefficient of friction decreases while maintaining its horizontal position on sliding. If the lateral force occurred after the beginning of the braking of the vehicle (braking before the turn, etc.), then the maximum of the friction coefficient reduces when shifting to the left on the sliding.

REFERENCES


